Final Review: Chapters 1-11, 13-14

These are selected problems that you are to solve independently or in a team of 2-3 in order to better prepare for your Final Exam

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Problem 1: Chasing a motorist

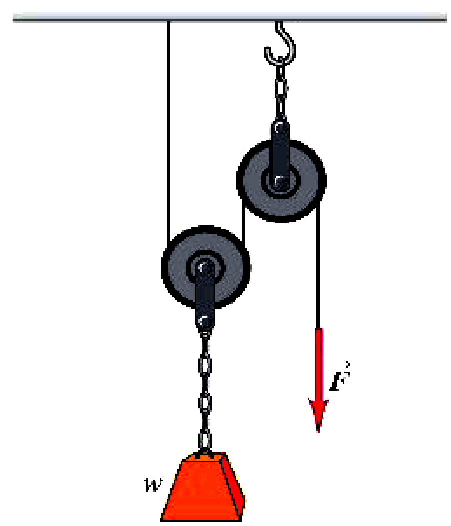
This is a 1-dimensional problem. A College Station police officer on a motorcycle is hiding behind a bush by a stop sign. A motorist driving at a constant velocity runs through the stop sign without slowing down and continues to drive at that constant velocity $\mathbf{v_o}$. Right at the moment the motorist passes through the stop sign, the officer begins to chase the motorist with a constant acceleration $\boldsymbol{a}$.

a) In terms of $\mathbf{v_o}$ and/or $\boldsymbol{a}$, at what distance from the stop sign will the officer catch up with the motorist?
b) In terms of $\mathbf{v_o}$ and/or $\boldsymbol{a}$, what is the speed of the officer the moment he catches up with the motorist?

Problem 2: Weight Lifting

A worker lifts a weight w = 900 N by pulling down on a rope with a force F = 600 N, as shown in the diagram. Assume that the rope and pulleys are massless. Ignore all frictions. The rope is un-stretchable. Find

a) The acceleration of the weight;
b) The work done by the worker for lifting the weight by 1 m.
c) The force required for keeping the weight at rest?



Problem 3: Small box sliding

This is a 1-dimensional problem. A small box of mass M has an initial speed v_o. It comes to a complete stop after sliding a distance L on a leveled surface with an unknown coefficient of kinetic friction. Neglect any spinning (rotations) of this small box.

a) Draw the free body diagram for the box.
b) Find the acceleration of the box.
c) Find the coefficient of kinetic friction between the box and the level surface.
d) Find the velocity of the box at the moment when it has moved a distance L/2.
e) Find the rate of energy dissipation at the moment when the box has moved a distance L/2.

Problem 4: Rolling Cylinder

A cylinder of radius R and mass M is rolling down an incline making an angle q with respect to the horizontal. What is the minimum static friction coefficient m_s that is required to have rolling without slipping?

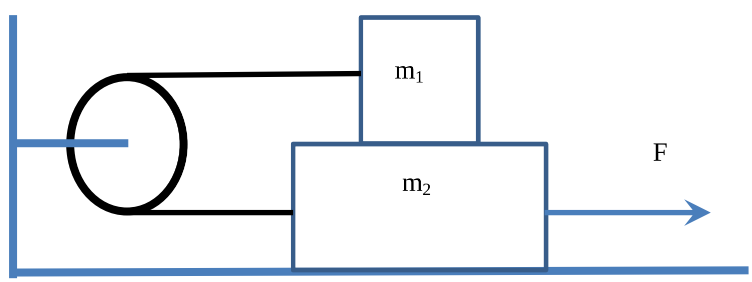
Problem 5: Pendulum

A small glue ball is fired along the horizontal into the lower end of a uniform rod, pivoted at its upper end. After the impact, the ball is stuck to the rod and the rod swings up to an angle $\boldsymbol{\beta}$. The mass of the glue ball is $\boldsymbol{m}$. The rod has mass $\boldsymbol{M}$ and length $\boldsymbol{L}$. Ignore friction and air resistance.

a) With the glue ball stuck to the lower end of the rod, calculate the moment of inertia of the combined object for an axis through the pivotal point and perpendicular to the plane of rotation.
b) Find the position of the center of mass for the combined object.
c) Find the speed of the ball before the impact.
d) Derive the pendulum's oscillator equation for the combined object if the angle of deflection is very small.
e) Find the frequency of the small oscillations of the combined object.

Problem 6: Stacked boxes

As shown in the drawing, the Box1 of mass m_1 is stacked on top of the Box2 mass m_2 on a leveled table surface. The two boxes are connected to each other with a massless and non-stretchable rope, which goes around a frictionless and massless pulley that can rotate freely about the axis through its center. We assume that the friction forces between Box1 and Box2 and between Box2 and the table surface have the same kinetic and static friction coefficient, m. A horizontal force F is applied on Box2 to pull it to move to the right. Set up a coordinate system such that the x-axis points to the right horizontally and the y-axis points up vertically.

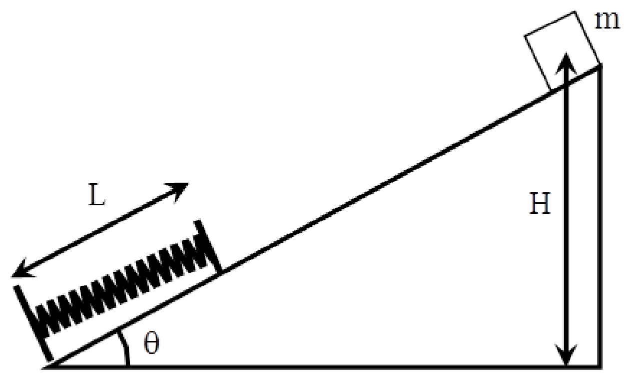


a) Draw one free-body diagram for each box.
b) Write down the Newton's Law equation(s) for Box1 based on its free-body diagram.
c) Write down the Newton's Law equation(s) for Box2 based on its free-body diagram.
d) What is the horizontal force F that is required to pull Box2 at a constant velocity?
e) If the horizontal force applied to Box2 is larger than the value found in (d), calculate the acceleration of Box2.

Problem 7: Firing spring

A massless spring has a relaxed length L. As shown in the drawing, it is placed on an inclined surface making an angle q with respect to the horizontal. The lower end of the spring is fixed to the bottom of the incline, and the upper end is free to move along the incline. The spring has a spring constant k. A small box of mass m is initially held at rest at a height H. After it is released, the box slides down the incline, makes contact with the upper end of the spring, sticks to it, and compress the spring by a certain length x. Assume that the coefficient of kinetic friction between the box and the inclined surface is m.

a) Set up a suitable coordinate system and draw it on the sketch.
b) Given the amount of compression x, calculate:
    i. the work done by the spring restoring force
    ii. the work done by the weight of the box
    iii. the work done by the friction force
c) Combining the results you obtain for part (b), obtain a relationship (or equation) which can be used to calculate the maximum amount of compression x_m based on all the other given parameters. You do not need to solve for x_m.



Problem 8: Explosion

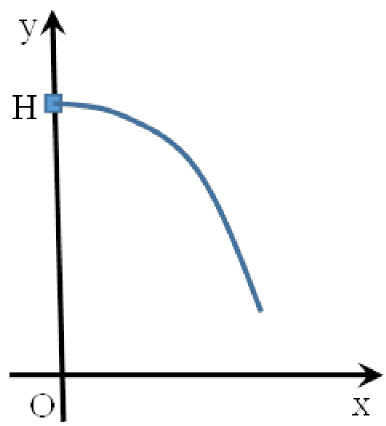
A small object, moving on a leveled frictionless surface, initially had a constant velocity of an unknown magnitude. It then splits into two parts which start moving perpendicular to each other. These two parts have known masses m_1 and m_2, and known speeds v_1 and v_2, respectively.

a) Sketch a diagram and a coordinate system suitable for solving this problem.
b) Calculate the magnitude of the initial velocity of the object before splitting.

Problem 9: Flying package

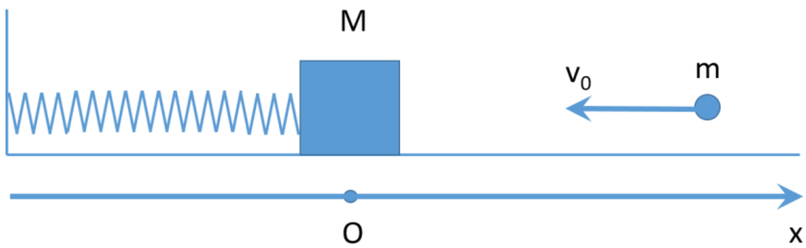
A cargo plane flies with a constant velocity v_0 at a constant altitude H above the ground. Set up a rectangular coordinate system fixed on the ground, with the x-axis pointing in the direction of the airplane velocity and the y-axis pointing vertically up like in the figure below. At the moment t = 0, when the airplane flies right above the origin, it releases a package which experiences a free-fall (neglect air friction). Take into account only the time interval from the moment of release until the package hits the ground.

a) Calculate the coordinates of the package at the moment *t*: x(t) and y(t).
b) Calculate the angular momentum of the package with respect to the origin of the system of coordinates at the moment of time *t*.
c) Calculate the torque of on the package at the moment of time *t*. Choose the pivot at the origin of the system of coordinates.
d) Does the motion of the package obey Newton's second law for rotational motion?



Problem 10: **Shooting a Springy Box**

A massless spring with a spring constant ***k*** is loaded with a block of mass ***M*,** resting in equilibrium on a frictionless leveled surface. A bullet of mass ***m***, with a horizontal velocity $\mathbf{v_o}$ as indicated in the sketch, strikes the block at time t = 0 and remains inside the block.

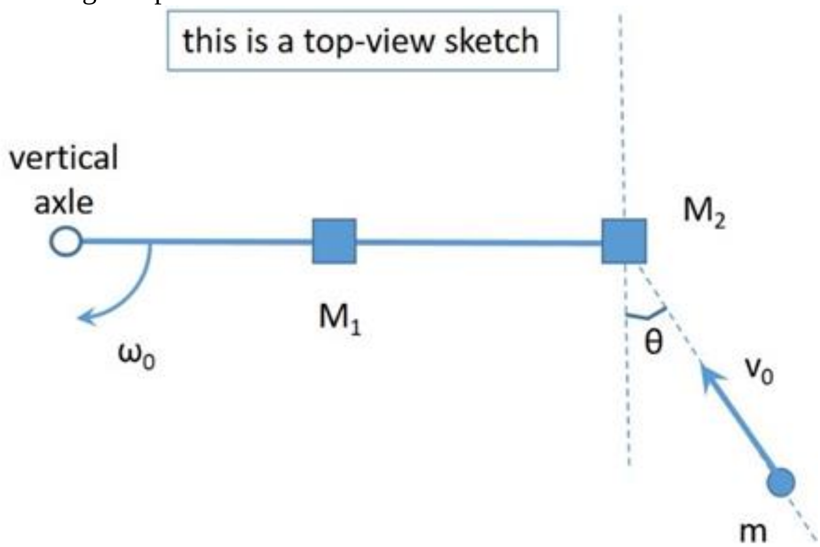


a) Calculate the amount of the maximum compression of the spring.
b) What is the position x(t) of the block as a function of time?
c) What is the frequency of the oscillation?
d) At what time does the block make its first return to the equilibrium position
e) At what position does the block has the maximum speed?

Problem 11: Shooting a Rotating System

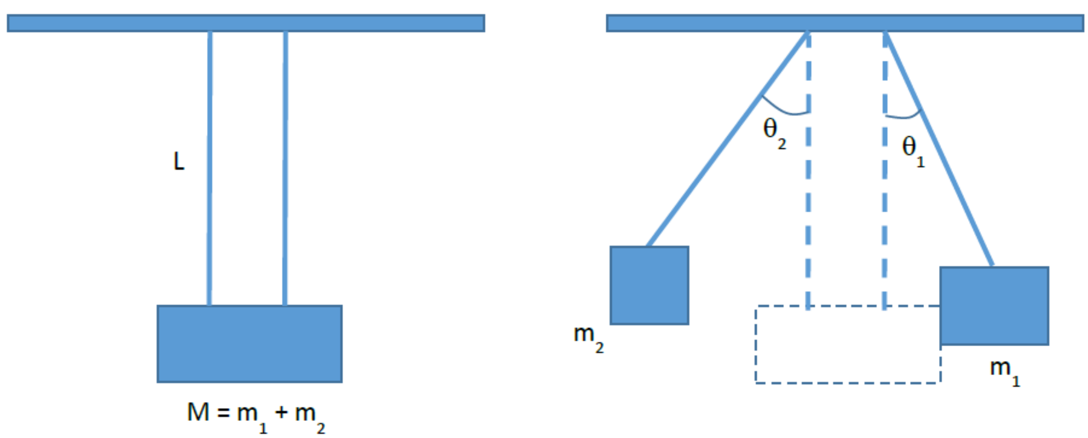
Ignore gravity in this problem. As shown in the top-view sketch, one end of a rod of length B is attached to a vertical axle. The rod is free to rotate about the axle in a horizontal plane, with a moment of inertia I. Two small blocks of masses M_1 and M_2 are located on the rod at distances A and B from axle, respectively. The rod is initially set to spin at an angular velocity of magnitude ω_o as shown in the sketch. A bullet of mass m and velocity v_o moving horizontally strikes the block of mass M_2 at the given angle θ. The bullet stops inside the block.

a) Calculate the angular velocity of the rod after the bullet struck the block.
b) Comment on how the direction of the system angular momentum after the collision depends on the given parameters.



Problem 12: Explosion in a box

A box of mass M is hung from the ceiling by strings of length L as shown. At one particular moment of time, it suddenly splits into two pieces of known masses m_1 and m_2. Right after they split, these two pieces move in opposite directions with unknown speeds v_1 and v_2, respectively. It is measured that m_1 swings up to an angle θ_1 as shown Assume that none of the pieces rotates about its center of mass. What angle should m_2 swing up to?



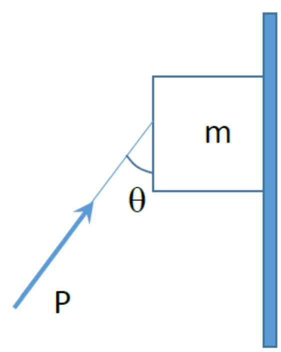
Problem 13: Satellite

A ***geosynchronous satellite*** has the orbital period equal to Earth's rotation period. As such, it appears to be "fix" in the sky. These geosynchronous satellites are critically important for GPS navigation. Assuming that the Earth is spherical with a radius of 6370 km and a mass of 5.97×10^{24} kg, calculate:

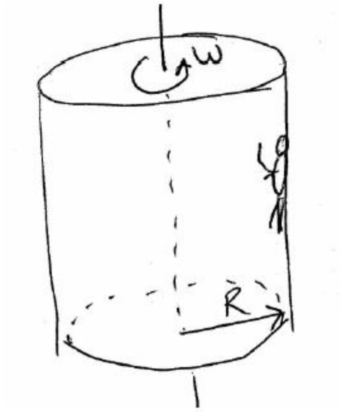
a) The altitude of such a satellite.
b) The orbital speed of such a satellite.

Problem 14: Stuck on a fixed Wall

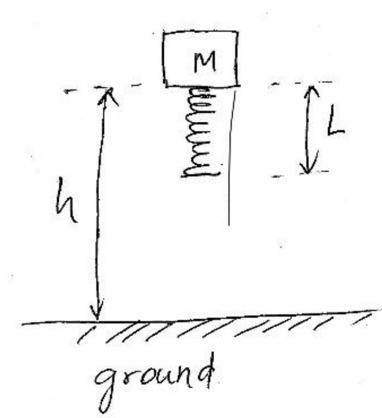
A small block of mass m = 20 kg is pushed against a vertical wall by a force P as shown in the figure aside. The angle is $\theta = 30^{o}$ and the static friction coefficient is μ_s. What is minimum value of the force P that keeps the block at rest?



Problem 15: Stuck on a Rotating Wall
In an amusement park, you are backed to a vertical wall of a circular room that is pitch-dark. Then, everything starts to spin. Even worse, at some point of time, the floor below your feet starts to sink. Given your mass m, the static friction coefficient μ_s between you and the wall, the radius R of the spinning room, calculate the minimum angular velocity ω of the spinning room that is required for the wall to be able to hold you up.

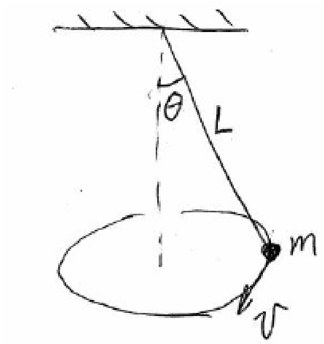


Problem 16: Jack in a box
This is a 1-dimensional problem. Imagine that you attach a spring to the bottom of a box of mass ***M*** as shown in the figure. The length of the spring is ***L*** and the spring constant is ***k***. You initially hold the box at rest at a height ***h*** above the ground then you release it to fall freely. Assume that there is no rotational motion and the spring hits the ground elastically.
By how much the spring should compress when the box reaches the lowest point?



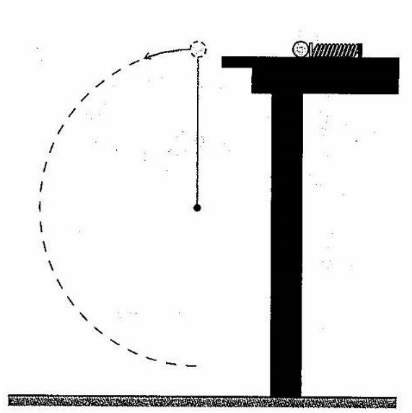
Problem 17: Conical Pendulum

A conical pendulum consists of a point particle of mass *m* hung from a fixed point with a massless and un-stretchable string of length ***L***. Imagine that you want the point mass to maintain a uniform circular motion in a horizontal plane with the string making an angle ***θ*** with respect to the vertical. What speed should the point mass have?



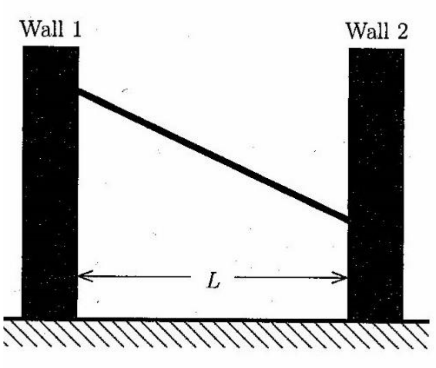
Problem 18: Ball Catcher

A ball that weighs 25 N is placed on a smooth horizontal table in front of a spring that has a force constant ***k***. When the ball and the spring are compressed and released, the ball shoots off the table and is caught on the end of a 3.00-m-long massless string. The ball then moves in a circular path as shown in the figure below. The string doesn't break unless the spring is compress more than 1.00 m, and the maximum tension the string can sustain is 200 N. What is the value of the spring constant? (Ignore the fact that it would be odd for a string to be standing straight up as shown).



Problem 19: Turnpike

Two vertical walls are separated by a distance L as shown in the figure. Wall1 is smooth, while Wall2 is not. A uniform board is propped between them. The coefficient of static friction between the board and Wall2 is μ. What is the length of the longest board that can be propped between the walls without slipping?



Problem 19: Satellite changing orbit

A 1000kg-satellite is in uniform circular motion on a 200km-orbit above the surface of the Earth. How much energy must be supplied to this satellite to move it on a circular at a height of 3000 km above the Earth? Does the period of revolution of this satellite change when changing the orbit? If yes, what would be the ratio of the new period to the old period? Justify your answer with appropriate diagrams and equations.